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RELATIONS OF THE THEORY OF THIN TIMOSHENKO-TYPE SHELLS IN TERMS OF THE CURVILINEAR COORDINATES OF THE REFERENCE SURFACE

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A method is given for solving the problem of parametrization and determination of the metric of the middle surface of a shell of complex configuration. The method is based on introducing, into the space, a surface σ_0 with simple geometry, and using this as a reference surface on which the middle surface σ is mapped. The position of a point on σ is defined in terms of the Gaussian coordinates α^1, α^2 of the point on σ_0, and the distance $H(\alpha^1, \alpha^2)$ between σ and σ_0 measured along the normal to σ_0. Expansion of the shell displacement vector in terms of the basis vectors on σ, the basis representing a mapping of the basis on σ_0, and use of a Timoshenko-type theory, yield a formulation of a nonlinear boundary value problem of computing shells of complex configuration. A method is given of reducing the problem of investigating the open "non-classical" shells in terms of the coordinates of their middle surfaces to a "conditionally classical" problems in terms of the coordinates of the reference surface. A theory of shells shallow relative to the reference surface is proposed, which generalizes the classical theory of shallow shells the middle surface of which is shallow relative to a plane.

1. **Mapping of the middle surface of the shell onto the reference surface.** We know that when a continuous 1:1 correspondence is established between the points of two surfaces σ_0 and σ, then each of the two corresponding points can be assigned the same values of the curvilinear coordinates. Such a parametrization of the surface parameters is called general with respect to the correspondence in question. The equations of the surfaces taking part in the general parametrization have the form $\mathbf{r}^\circ = \mathbf{r}^\circ(\alpha^1, \alpha^2)$, $\mathbf{r} = \mathbf{r}(\alpha^1, \alpha^2)$.

The coordinate surface in the theory of shells is normally made to coincide with the middle surface by assuming that $\mathbf{r}^\circ(\alpha^1, \alpha^2) = \mathbf{r}(\alpha^1, \alpha^2)$, and this causes considerable difficulties in solving the problems of parametrization and determination of the metric of the middle surface of a shell of complex configuration. Let σ denote the middle surface of the undeformed shell. We introduce a surface σ_0 specified by its lines of curvature α^1, α^2, and call it from now on the reference surface. We shall use the notation and auxilliary formulas of [1].

If $\mathbf{r}^\circ$ is the radius vector of a point $M_0 \in \sigma_0$ and $\mathbf{m}^\circ$ is the unit vector normal to σ_0 at this point, then, choosing the form and position of the point σ_0 in the space in the appropriate manner, we can determine the position of a point $M \in \sigma$ from the vector equation

$$\mathbf{r}(\alpha^1, \alpha^2) = \mathbf{r}^\circ(\alpha^1, \alpha^2) + H(\alpha^1, \alpha^2)\mathbf{m}^\circ \tag{1.1}$$

where H is the distance between σ_0 and σ. Clearly the mapping (1.1) will be in 1:1 correspondence if a straight line drawn along the normal to σ_0 from each point of σ_0 intersects σ not more than once. With such a method of parametrization of the middle surface, the coordinate lines $\beta^i \in \sigma$ will be the lines drawn by the tip of the radius vector $\mathbf{r}$ when the point M_0 moves along the coordinate lines $\alpha^i \in \sigma_0$.

Differentiating (1.1) with respect to α^i and taking into account the formulas $\mathbf{m}_{,i}^\circ = A_i^\circ k_i \mathbf{e}_i^\circ$, we find the coordinate vectors of the fundamental basis of σ and the components of the metric tensor (k_i are the curvatures of the coordinate lines $\alpha^i \in \sigma_0$)

$$\mathbf{r}_i = A_i^\circ \theta_i (\mathbf{e}_i^\circ + y_i \mathbf{m}^\circ), \quad \mathbf{e}_i^\circ = \mathbf{r}_i^\circ / A_i^\circ \tag{1.2}$$

$$\begin{aligned} a_{ik} &= \mathbf{r}_i \mathbf{r}_k = A_i^\circ A_k^\circ \theta_i \theta_k (\delta_{ik} + y_i y_k) \\ A_i^\circ &= |\mathbf{r}_i^\circ|, \quad \theta_i = 1 + Hk_i, \quad y_i = H_{,i} / (A_i^\circ \theta) \end{aligned} \tag{1.3}$$

Let us write the unit vector $\mathbf{m}$ normal to σ in the form of an expansion

$$\mathbf{m} = \xi\, (\mathbf{m}^\circ = \xi_i \mathbf{e}_i^\circ) \tag{1.4}$$

where ξ, ξ_1 and ξ_2 are unknown coefficients. Substituting (1.2) and (1.4) into the scalar products $\mathbf{mm} = 1$, $\mathbf{mr}_i = 0$, we find

$$\xi_i = y_i, \quad \xi = (1 + y_1^2 + y_2^2)^{-1/2} \tag{1.5}$$

and substituting the formulas (1.2) and (1.4) into the expressions for b_{ik}, we obtain

$$\begin{aligned} b_{11} &= -A_1^\circ \theta_1 \xi (A_1^\circ k_1 C_1^2 + y_2 A_{1,2}^\circ / A_2^\circ - y_{1,1}) \quad (1,2) \\ b_{12} &= -A_2^\circ \theta_2 \xi (y_1 y_2 A_1^\circ k_1 - y_{2,1} + y_1 A_{1,2}^\circ / A_2^\circ) = b_{21}, \quad C_i^2 = 1 + y_i^2 \end{aligned} \tag{1.6}$$

Here and henceforth the symbol (1, 2) indicates that the remaining relations are obtained by interchanging the indices 1 and 2 in the expressions given.

Thus the relations (1.3) and (1.6) enable us to determine, in a sufficiently simple manner, the metric of the middle surface of the shell σ, provided that the metric of the reference surface and the distance between σ and σ_0 are both known.

2. Relations of the theory of mean flexure of thin, Timoshenko-type shells in terms of the curvilinear coordinates of the reference surface. In studying the stress-strain states of thin shells by numerical methods, it is expedient to make use of the relations of the theory of Timoshenko-type shells based on the straight line hypothesis. According to this hypothesis the displacement vector of a point P on the shell situated, before the deformation, at a distance z from σ, can be written in the form

$$\boldsymbol{\omega}^z = \mathbf{v} + z\boldsymbol{\gamma} = u_i \mathbf{r}^i + w\mathbf{m} + z(\gamma_i \mathbf{r}^i + \gamma \mathbf{m}), \quad -h/2 \leqslant z \leqslant h/2 \tag{2.1}$$

Here u_i and γ_i are the covariant components of the displacement vector of the middle surface σ and of the rotation vector $\mathbf{r}^i = a^{ik}\mathbf{r}_k$ are the mutual basis vectors and h is the shell thickness. It can be shown that

$$a^{11} = (\xi C_2 / A_1{}^\circ\theta_1)^2, \quad a^{22} = (\xi C_1 / A_2{}^\circ\theta_2)^2, \quad a^{12} = -\xi y_1 y_2 / A_1{}^\circ A_2{}^\circ\theta_1\theta_2$$

The radius vectors of the point P before and after deformation are $\mathbf{R} = \mathbf{r} + z\mathbf{m}$, $\mathbf{R}^* = \mathbf{R} + \mathbf{V}^z$. Differentiating these expressions with respect to α^i and taking into account (2.1), we obtain the basis vectors (the notation of [2] is used here)

$$\begin{aligned}
&\mathbf{R}_i = (\delta_i{}^k - zb_i{}^k)\mathbf{r}_{ik}, \quad \mathbf{R}_3 = \mathbf{m}, \quad \mathbf{R}_i{}^* = \mathbf{R} + \partial_i \mathbf{v} + z\partial_i \boldsymbol{\gamma} \\
&\mathbf{R}_3{}^* = \mathbf{m} + \boldsymbol{\gamma}, \quad \partial_i \mathbf{v} = e_{ik}\mathbf{r}^k + \omega_i \mathbf{m}, \quad \partial_i \boldsymbol{\gamma} = \Omega_{ik}\mathbf{r}^k + \Omega_i \mathbf{m} \\
&e_{ik} = \nabla_i u_k - b_{ik} w, \quad \omega_i = \nabla_i w + u_k b_i{}^k \\
&\Omega_{ik} = \nabla_i \gamma_k - b_{ik}\gamma, \quad \Omega_i = \nabla_i \gamma + \gamma_k b_i{}^k
\end{aligned} \tag{2.2}$$

Let us quote some of the relations of the theory of mean flexure [1, 2] for the case of small transverse displacements. We call the flexure of the shell mean, if its maximum value is of the same order as the thickness h. It was shown in [2] that the mean flexure has the following corresponding deformation components:

$$\begin{aligned}
&2\varepsilon_{ik} = e_{ik} + e_{ki} + \omega_i\omega_k, \quad 2\varepsilon_{i3} = \omega_i + \gamma_i \\
&2\varkappa_{ik} = \Omega_{ik} + \Omega_{ki}, \quad 2\varepsilon_3 = 2\gamma + \gamma_i\gamma^i, \quad \Omega_{ik} = \nabla_i\gamma_k
\end{aligned} \tag{2.3}$$

where ε_{ik} and $\varkappa_{ik}$ denote the covariant components of the tangential and bending deformation tensors and $2\varepsilon_{i3}$ are the transverse shears undergoing no change across the thickness of the shell.

In the Timoshenko-type shear model the deformation components are

$$\varepsilon_{ik}{}^z = \varepsilon_{ik} + z\varkappa_{ik}, \quad \varepsilon_{i3}{}^z = \varepsilon_{i3} \tag{2.4}$$

The equations of equilibrium corresponding to the deformation components (2.3) can be obtained from the variational Lagrange equation ($\sigma^{33} = 0$, δA denotes the elementary work of the external forces and moments [1])

$$\begin{aligned}
\delta A &= \iint_{\sigma_0} \int_{-h/2}^{h/2} (\sigma^{ik}\delta\varepsilon_{ik}^z + 2\sigma^{i3}\delta\varepsilon_{i3})\, d\sigma\, dz = \\
&\iint_{\sigma_0} [T^{ik}(\delta e_{ik} + \omega_i\delta\omega_k) + M^{ik}\delta\Omega_{ik} + N^i(\delta\omega_i + \delta\gamma_i)]\, d\sigma \\
\delta A &= \iint_{\sigma_0} (\mathbf{X}\delta\mathbf{v} + \mathbf{M}\delta\boldsymbol{\gamma})\, d\sigma + \int_C (\boldsymbol{\Phi}^s\delta\mathbf{v} + \mathbf{M}^s\delta\boldsymbol{\gamma})\, ds
\end{aligned} \tag{2.5}$$

where we have put $\delta_i{}^k - zb_i{}^k \approx \delta_i{}^k$ and defined the forces and moments as follows:

$$T^{ik} = \int_{-h/2}^{h/2} \sigma^{ik}\, dz, \quad M^{ik} = \int_{-h/2}^{h/2} \sigma^{ik} z\, dz, \quad N^i = \int_{-h/2}^{h/2} \sigma^{i3}\, dz \tag{2.6}$$

The variational formula (2.5) can be transformed [1] to the form

$$\int\limits_C [(\Phi - \Phi^s)\,\delta\mathbf{v} + (\mathbf{M} - \mathbf{M}^s)\,\delta\boldsymbol{\gamma}]\,ds - \iint\limits_{\sigma_0} (L^i\delta u_i + L^3\delta w + H^i\delta\gamma_i)\,d\sigma = 0 \quad (2.7)$$

and this yields the following equations of equilibrium and the static boundary conditions:

$$L^i = \nabla_k T^{ik} - b_k{}^i N^{k3} + X^i = 0, \quad L^3 = \nabla_k N^{k3} + b_{ik} T^{ik} + X^3 = 0 \quad (2.8)$$
$$H^i = \nabla_k M^{ik} - N^i + M^i = 0, \quad N^{i3} = N^i + T^{ik}\omega_k'$$

$$\Phi = \Phi^s, \quad M = M^s \quad (2.9)$$

The above method of investigating shells of complex configuration represents a novel approach to the problem of solving a wide class of open shells in which the normal projection of the contour $C \in \sigma$ on σ_0 coincides with the coordinate lines $\alpha^i = 0$, $a_i \in \sigma_0$. A typical example belonging to this class is a shell cut out of a shell of revolution by the sections $x_1 = 0, h_1$ $x_2 = h_2$ and shown in Fig. 1. The shell is non-classical since its two contour lines do not coincide with the meridional lines of the surface of revolution. It can be however reduced to the classical form by choosing, in accordance with the above statements, a circular cylindrical surface of radius R_0 with the axis laying in the plane $x_2 = h_2$ and parallel to the axis of the surface of revolution.

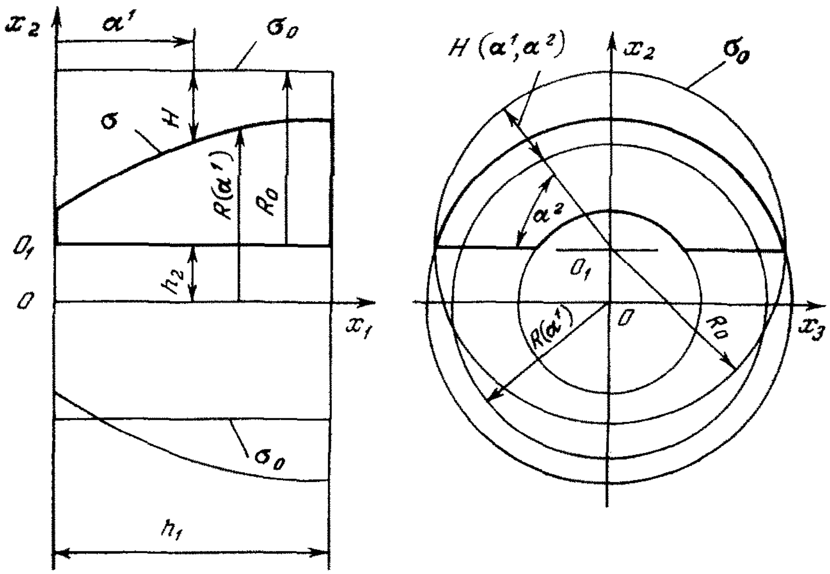

Fig. 1

Let us derive the boundary conditions for the class of shells in question. Remembering that on the coordinate lines $\beta^1 = \text{const} \in C$ which are a mapping of the coordinate lines $\alpha^1 = 0, a_1$, the vectors $\mathbf{r}_2$ are tangential and $\mathbf{r}^1$ are normal to the contour C, we write the tangential displacement vectors $\mathbf{v}^t$ and rotational displacement vectors γ^t in the form $\mathbf{v}^t = u_\tau \mathbf{r}_2 + u_n \mathbf{r}^1$, $\gamma^t = \gamma_\tau \mathbf{r}_2 + \gamma_n \mathbf{r}^1$. Let us write the covariant components of these vectors in terms of u_τ, u_n, γ_τ and γ_n, constructing the vector relations

$$u_\tau \mathbf{r}_2 + u_n \mathbf{r}^1 = u_i \mathbf{r}^i, \quad \gamma_\tau \mathbf{r}_2 + \gamma_n \mathbf{r}^1 = \gamma_i \mathbf{r}^i \tag{2.10}$$

Scalarly multiplying (2.10) by $\mathbf{r}_1$ and then by $\mathbf{r}_2$, we find

$$\begin{aligned} u_1 &= u_\tau a_{12} + u_n, \quad u_2 = u_\tau a_{22} \\ \gamma_1 &= \gamma_\tau a_{12} + \gamma_n, \quad \gamma_2 = \gamma_\tau a_{22} \quad (1,\ 2) \end{aligned} \tag{2.11}$$

The contour integral given in (2.7) can be transformed, in the present case, with (2.11) taken into account, to

$$\begin{aligned} &\int_C [(\boldsymbol{\Phi} - \boldsymbol{\Phi}^s)\,\delta\mathbf{v} + (\mathbf{M} - \mathbf{M}^s)\,\delta\gamma]\,ds = \int_0^{a_2} \Lambda_{12}\,d\alpha^2 \Big|_0^{a_1} + \int_0^{a_1} \Lambda_{21}\,d\alpha^1 \Big|_0^{a_2} \\ &\Lambda_{12} = (\boldsymbol{\Phi}^1 \sqrt{a} - \boldsymbol{\Phi}_s^1 A_2)\delta\mathbf{v} + (\mathbf{M}^1 \sqrt{a} - M_s^1 A_2)\delta\gamma = \\ &\quad [(T^{11} a_{12} + T^{12} a_{22})\sqrt{a} - (\Phi_s^{11} a_{12} + \Phi_s^{12} a_{22}) A_2]\delta u_\tau + \\ &\quad (T^{11} \sqrt{a}_{12} - \Phi_s^{11} A_2)\delta u_n + (N^{13}\sqrt{a} - \Phi_s^{13} A_2)\delta w + \\ &\quad [(M^{11}\sqrt{a} + M^{12} a_{22})\sqrt{a} - (M_s^{11} a_{12} + M_s^{12} a_{22}) A_2]\delta\gamma_\tau + \\ &\quad (M^{11}\sqrt{a} - M_s^{11} A_2)\delta\gamma_n \quad (1,\ 2) \\ &(A_i = \sqrt{a_{ii}}, \ \boldsymbol{\Phi}_s^i = \Phi_s^{ik}\mathbf{r}_k + \Phi_s^{i3}\mathbf{m}, \quad \mathbf{M}_s^i = \mathbf{M}_s^{ik}\mathbf{r}_k + M_s^{i3}\mathbf{m}) \end{aligned}$$

Consequently, various variants of the static and geometrical boundary conditions on the contour C can be assembled, for $\alpha^i = 0, a_i$, from

$$\begin{aligned} &(T^{11} a_{12} + T^{12} a_{22})\sqrt{a} - (\Phi_s^{11} a_{12} + \Phi_s^{12} a_{22}) A_2 = 0, \quad \delta u_\tau = 0 \\ &T^{11}\sqrt{a} - \Phi_s^{11} A_2 = 0, \quad \delta u_n = 0 \\ &N^{13}\sqrt{a} - \Phi_s^{13} A_2 = 0, \quad \delta w = 0 \\ &(M^{11} a_{12} + M^{12} a_{22})\sqrt{a} - (M_s^{11} a_{12} + M_s^{12} a_{22}) A_2 = 0, \quad \delta\gamma_\tau = 0 \\ &M^{11}\sqrt{a} - M_s^{11} A_2 = 0, \quad \delta\gamma_n = 0 \quad (1,\ 2) \end{aligned} \tag{2.12}$$

3. Relations of the theory of thin shells shallow relative to the reference surface. We shall consider a particular case in which the conditions of shallowness of the middle surface of the shell with respect to the reference surface $y_i y_k = (A_i^\circ A_k^\circ \theta_i \theta_k)^{-1} H_{,i} H_{,i} \ll 1$; $i, k = 1, 2$ can be imposed on the variation in the quantity $H(\alpha^1, \alpha^2)$. As a result of these conditions, we can introduce a number of simplifications in what follows. First we have, with the accuracy of $1 + y_i^2 \approx 1$,

$$C_i = (1 + y_i^2)^{1/2} \approx 1, \quad \xi = (1 + y_1^2 + y_2^2)^{-1/2} \approx 1 \tag{3.1}$$

Moreover, the coordinate lines β^i and σ can be regarded as orthogonal assuming that

$$\cos\chi = y_1 y_2 / (C_1 C_2) \approx 0, \quad \sin\chi = 1 / (C_1 C_2 \xi) \approx 1 \tag{3.2}$$

where χ is the angle between the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$.

It follows that the fundamental basis of the surface σ can be assumed to coincide with the mutual basis. Taking into account (1.5), (3.1) and (3.2) we obtain from (1.2) and (1.4)

$$\mathbf{e}_i = \mathbf{r}_i / A_i \approx \mathbf{e}_i^\circ + y_i \mathbf{m}^\circ, \quad \mathbf{m} \approx \mathbf{m}^\circ - y_1 \mathbf{e}_1^\circ - y_2 \mathbf{e}_2^\circ \tag{3.3}$$

and write the formulas for the curvatures k_{ik} of the coordinate lines $\beta^i \in \sigma$ with help of the Codazzi conditions $(k_2 A_2^\circ)_{,1} = k_1 A_{2,1}^\circ$ (1, 2), in the form

$$\begin{aligned} k_{11} &= -b_{11} / a_{11} = k_1 / \theta_1 - (A_2^\circ y_{1,1} + y_2 A_{1,2}^\circ) / A_1^\circ A_2^\circ \\ A_1 A_2 k_{12} &= -b_{12} = \theta_2 y_1 A_{1,2}^\circ + \theta_1 y_2 A_{2,1}^\circ - H_{,12} \quad (1,2) \end{aligned} \tag{3.4}$$

where by virtue of $C_i \approx 1$ the coefficients $A_i \approx A_i^\circ \theta_i$.

Finally, taking into acccount (3.1) and (3.2) and using the Codazzi conditions, we obtain the approximate differentiation formulas

$$\begin{aligned} A_2 \mathbf{e}_{1,1} &= -A_{1,2} \mathbf{e}_2 - A_1 A_2 k_{11} \mathbf{m}, \quad A_1 \mathbf{e}_{1,2} = A_{2,1} \mathbf{e}_2 - A_1 A_2 k_{12} \mathbf{m} \\ \mathbf{m}_{,1} &= A_1 (k_{11} \mathbf{e}_1 + k_{12} \mathbf{e}_2) \quad (1, 2) \end{aligned} \tag{3.5}$$

which coincide formally with the corresponding formulas of [1].

Let us denote by u_i, w and γ_i the physical components of the vectors $\mathbf{v}$ and γ: $\mathbf{v} = u_i \mathbf{e}_i + w \mathbf{m}$, $\gamma = \gamma_i \mathbf{e}_i + \gamma \mathbf{m}$. Then, using the formulas (3.3) and (3.5) we obtain, with the accuracy of $1 + y_i y_k \approx 1$, the physical components of the deformation tensor

$$\begin{aligned} 2\varepsilon_{ik} &= e_{ik} + e_{ki} + \omega_i \omega_k, \quad 2\varepsilon_{i3} = \omega_i + \gamma_i \\ 2\varkappa_{ik} &= \Omega_{ik} + \Omega_{ki} \end{aligned} \tag{3.6}$$

where in contrast to (2.3)

$$\begin{aligned} e_{11} &= A_1^{-1} u_{1,1} + u_2 A_{1,2} (A_1 A_2)^{-1} + k_{11} w, \quad e_{12} = A_1^{-1} u_{2,1} - \\ &\quad u_1 A_{1,2} (A_1 A_2)^{-1} + k_{12} w, \quad \omega_1 = A_1^{-1} w_{,1} - k_{11} u_1 - k_{12} u_2 \\ \Omega_{11} &= A_1^{-1} \gamma_{1,1} + \gamma_2 A_{1,2} (A_1 A_2)^{-1}, \quad \Omega_{12} = A_1^{-1} \gamma_{2,1} - \gamma_2 A_{1,2} (A_1 A_2)^{-1} \quad (1, 2) \end{aligned} \tag{3.7}$$

The equations of equilibrium of the shell (2.8) can also be reduced with the accuracy of $1 + y_i y_k \approx 1$ to the form

$$\begin{aligned} L^1 &= (A_2 T_{11})_{,1} + [(A_1 T_{12})_{,2} + T_{12} A_{1,2} - T_{22} A_{2,1} + A_1 A_2 (N_{i3} k_{1i} + X_1) = 0 \\ L^3 &= (A_2 N_{13})_{,1} + (A_1 N_{23})_{,2} - A_1 A_2 (T_{ik} k_{ik} - X_3) = 0 \\ H^1 &= (A_2 M_{11})_{,1} + (A_1 M_{12})_{,2} + M_{12} A_{1,2} - M_{22} A_{2,1} + A_1 A_2 (M_1 - N_1) = 0 \\ &\qquad (1,2) \end{aligned} \tag{3.8}$$

where we have assumed that the expression $T_{12} = T_{21}$, $M_{12} = M_{21}$ is approximately true and introduced the notation $N_{i3} = N_i + T_{ik} \omega_k$. Here T_{ik}, M_{ik}, X_i, M_i, N_i denote the physical components of the corresponding tensors and vectors.

The boundary conditions (2.12) at the edges $\beta^i = \text{const}$, assume, by virtue of the relation $a_{12} \approx 0$, a very simple form

$$\begin{aligned} T_{ik} - \Phi_{ik}^s &= 0, \quad \delta u_k = 0 \\ N_{i3} - \Phi^s_{i3} &= 0, \quad \delta w = 0 \end{aligned} \tag{3.9}$$

$$M_{ik} = M_{ik}{}^s = 0, \quad \delta\gamma_k = 0, \quad i, k = 1, 2$$
$$(\Phi_i{}^s = \Phi_{ik}{}^s \mathbf{e}_k + \Phi_{i3}{}^s \mathbf{m}, \quad M_i{}^s = M_{ik}{}^s \mathbf{e}_k + M_{i3}{}^s \mathbf{m})$$

Let us call the relations (3.6) —(3.9) the relations of the theory of mean flexure for the Timoshenko-type shells shallow relative to the reference surface. The limits of applicability of these relations are determined by the limiting values of the angles between the coordinate vectors $\mathbf{r}_i$ and $\mathbf{r}_i{}^\circ$ and these, in turn, can be expressed in terms of the coefficients y_1 and y_2. Taking the admissible error in computing the basic factors determining the stress-strain state of the shell as $\varepsilon = 0.05$, we find that the limiting values of y_i are equal to ~ 0.225 and this is confirmed by the numerical example which was solved in [3].

We note that the equations (3.8) can be replaced by the approximate expressions

$$(A_2 T_{11})_{,1} + (A_1 T_{1,2})_2 + T_{12} A_{1,2} - T_{22} A_{2,1} + A_1 A_2 X_1 = 0 \quad (1,2)$$
$$L^3 = 0, \quad H^i = 0, \quad i = 1,2$$

when the shearing forces are neglected in the first two equations. The formulas for the rotations ω_i can also be replaced by $\omega_i = A_i^{-1} w_{,1}$ for the shells the middle surface of which is shallow with respect to a plane, or shallow with respect to any reference surface in the sense explained above and can be subdivided into a large number of parts shwllow in the classical sense.

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